Calculating K_{sp} from Solubility

The molar solubility of CaF_2 at 35°C is 1.24×10^{-3} M.

(a) What is the solubility of CaF_2 in g/L? A: 0.0968g/L

$$1.24 \times 10^{-3} \frac{\text{mol}}{\text{L}} \times \frac{78.0748\ \text{g}}{\text{mol}} = \boxed{0.0968\ \text{g/L}}$$

Molar Mass Ca 40.078 g/mol x1 = 40.078g/mol

F 18.9984 g/mol x2 = 37.9968g/mol

78.0748g/mol

(b) What is K_{sp} at this temperature? A: 7.63×10^{-9}

$$CaF_2(s) \rightleftharpoons Ca^{2+}(aq) + 2F^-(aq) \qquad K_{sp} = [Ca^{2+}][F^-]^2$$

$$X \qquad 2X$$

$[Ca^{2+}] = 1.24 \times 10^{-3}\ M$

$[F^-] = 2(1.24 \times 10^{-3}\ M) = 0.00248\ M$

$K_{sp} = [1.24 \times 10^{-3}][0.00248]^2 = \boxed{7.63 \times 10^{-9}}$

Calculating Solubility from K_{sp}

The K_{sp} for LaF_3 is $2.0x10^{-19}$.

(a) What is the molar solubility of LaF_3 in water? A: $9.3x10^{-6}$M

$$LaF_{3(s)} \rightleftharpoons La^{3+}(aq) + 3F^-(aq)$$

$$\quad\quad\quad\quad X \quad\quad\quad 3X$$

$$K_{sp} = [La^{3+}][F^-]^3$$

$$= [x][3x]^3 = 2.0 \times 10^{-19}$$

$$\frac{27x^4}{27} = \frac{2.0 \times 10^{-19}}{27}$$

$$x^4 = 7.4074 \times 10^{-21}$$

$$x = (7.4074 \times 10^{-21})^{1/4} = \boxed{9.3 \times 10^{-6} M}$$

(b) What is the solubility in g/L? A: $1.8x10^{-3}$g/L

$$\frac{9.3 \times 10^{-6} \text{ mol}}{L} \times \frac{195.9012 \text{ g}}{\text{mol}} = 1.8 \times 10^{-3} \text{ g/L}$$

La: 138.906 g/mol

F: 18.9984 g/mol x 3 = 56.9952 g/mol

$\overline{\quad 195.9012 \text{ g/mol}}$

Common-Ion Calculations

1. Calculate the pH of a solution containing 0.085M nitrous acid (HNO_2; $K_a = 4.5 \times 10^{-4}$) and 0.10M potassium nitrite (KNO_2).

	$HNO_2(aq)$	+ $H_2O(l)$ ⇌	$NO_2^-(aq)$	+ $H_3O^+(aq)$
I	0.085M		0.10M	0
C	−x		+x	+x
E	0.085 − x ignore		0.10 + x ignore	x

$$K_a = \frac{[a.10][x]}{[0.085]} = 4.5 \times 10^{-4}$$

$$\frac{[0.10][x]}{0.10} = \frac{3.825 \times 10^{-5}}{0.10}$$

$$x = 3.825 \times 10^{-4} = [H_3O^+]$$

$$pH = -\log(3.825 \times 10^{-4})$$

$$= \boxed{3.42}$$

A: pH = 3.42

2. The K_{sp} of $Mn(OH)_2$ is 1.6×10^{-13}. Calculate the molar solubility of $Mn(OH)_2$ in:

a.) water A: 3.4×10^{-5}M

b.) A solution that contains 0.020M NaOH A: 4.0×10^{-10}M

c.) Compare the solubility of $Mn(OH)_2$ in these solutions

A: 85,000 times more soluble in water

$Mn(OH)_2 \rightleftharpoons Mn^{2+}(aq) + 2OH^-(aq)$

X 2X

(a) $K_{sp} = [X][2X]^2 = 1.6 \times 10^{-13}$

$\frac{4X^3}{4} = \frac{1.6 \times 10^{-13}}{4}$

$X^3 = 4.0 \times 10^{-14}$

$X = (4.0 \times 10^{-14})^{1/3} = \boxed{3.4 \times 10^{-5}\ M}$

ignore

X $0.02M + 2X$

(b) $K_{sp} = [X][0.02]^2 = 1.6 \times 10^{-13}$

$\frac{[X][4.0 \times 10^{-4}]}{4.0 \times 10^{-4}} = \frac{1.6 \times 10^{-13}}{4.0 \times 10^{-4}}$ $\boxed{X = 4.0 \times 10^{-10}\ M}$

(c) $\frac{3.4 \times 10^{-5}\ M}{4.0 \times 10^{-10}\ M} = 85{,}000$ times more soluble in water than in 0.020M NaOH.

3.) How much is the solubility of lead (II) chloride changed in the presence of 0.85M NaCl? $K_{sp} = 1.6 \times 10^{-5}$

$PbCl_2(s) \rightleftharpoons Pb^{2+}(aq) + 2Cl^-(aq)$

$X \qquad 2X$

water: $K_{sp} = [X][2X]^2 = 1.6 \times 10^{-5}$

$\frac{4X^3}{4} = \frac{1.6 \times 10^{-5}}{4}$

$X^3 = 4.0 \times 10^{-6}$

$X = (4.0 \times 10^{-6})^{1/3} = 1.59 \times 10^{-2} M$

In 0.85M NaCl

$X \qquad 0.85 + \cancel{2X}$ (ignore)

$K_{sp} = [X][0.85]^2 = 1.6 \times 10^{-5}$

$\frac{[0.7225][X]}{[0.7225]} = \frac{1.6 \times 10^{-5}}{0.7225}$

$X = 2.2145 \times 10^{-5} M$

Compare: $\frac{1.59 \times 10^{-2} M}{2.2145 \times 10^{-5} M} = 718$

more than 700 times more soluble in water than in 0.85M NaCl

More than 700X less soluble than in water

Effect of pH Calculations

Calculate the solubility of $Mg(OH)_2$ (one of the ingredients in the antacid Maalox) in grams per liter when buffered at pH: (a) 12.50; (b) 7.00 A: a.) 7.01×10^{-7} g/L; b.) 7.00×10^{4} g/L

K_{sp} of $Mg(OH)_2 = 1.2 \times 10^{-11}$; MM $Mg(OH)_2 = 58.32$ g/mol

$$Mg(OH)_2 \rightleftharpoons Mg^{2+} + 2OH^-$$
$$\quad X \qquad 2X$$

$$K_{sp} = [x][2x]^2$$

(a) pH = 12.50 pOH = 14 − 12.50 = 1.5

$[OH^-] = 10^{-1.5} = 3.16 \times 10^{-2}$ M

$K_{sp} = [x][3.16 \times 10^{-2} + 2x]^2$ (ignore $2x$)

$$\frac{1.2 \times 10^{-11}}{9.9856 \times 10^{-4}} = \frac{[x][9.9856 \times 10^{-4}]}{9.9856 \times 10^{-4}}$$

$$X = 1.202 \times 10^{-8} \frac{mol}{L} \times \frac{58.32\,g}{mol}$$

$$X = 7.01 \times 10^{-7}\ g/L$$

(b) pH = 7 pOH = 14 − 7 = 7.00

$[OH^-] = 10^{-7} = 1.00 \times 10^{-7}$ M

$K_{sp} = 1.2 \times 10^{-11} = [x][1.00 \times 10^{-7} + 2x]^2$ (ignore $2x$)

$$1.2 \times 10^{-11} = [x][1.00 \times 10^{-7}]^2$$

$$\frac{1.2 \times 10^{-11}}{1.00 \times 10^{-14}} = \frac{[x][1.00 \times 10^{-14}]}{1.00 \times 10^{-14}}$$

$$X = 1.2 \times 10^{3} \frac{mol}{L} \times \frac{58.32\,g}{mol}$$

$$= \boxed{7.00 \times 10^{4}\ g/L}$$

Complex Ion Formation & Solubility

1. Determine the equilibrium constant for the reaction:

$$AgI(s) + 2CN^-(aq) \rightleftharpoons Ag(CN)_2^-(aq) + I^-(aq)$$

$$K_{sp}\ AgI = 8.3\times10^{-17};\ K_f\ Ag(CN)_2^- = 1\times10^{21}$$

$AgI(s) \rightleftharpoons Ag^+(aq) + I^-(aq) \quad K_{sp} = 8.3\times10^{-17}$

$Ag^+(aq) + 2CN^- \rightleftharpoons Ag(CN)_2^-(aq) \quad K_f = 1\times10^{21}$

$AgI(s) + 2CN^- \rightleftharpoons Ag(CN)_2^- + I^-(aq)$

$K_c = K_{sp} \times K_f$

$= (8.3\times10^{-17})(1\times10^{21}) = 8.3\times10^4$

$\hookrightarrow \boxed{8\times10^4}$

A: 8×10^4

Complex Ion Formation & Solubility

2. Calculate the molar solubility of zinc carbonate at 25˚C in (a) pure water and (b) 1.0M NH_3

$K_{sp} = 1.4\times10^{-11}$; $K_f = 4.1\times10^{8}$ for $Zn(NH_3)_4^{2+}$

A: (a) 3.7×10^{-6}M
(b) 7.6×10^{-2}M

$$ZnCO_3(s) \rightleftharpoons Zn^{2+}(aq) + CO_3^{2-}(aq)$$
X X

(a) = $K_{sp} = 1.4\times10^{-11} = [x][x]$

$x^2 = 1.4\times10^{-11}$

$x = (1.4\times10^{-11})^{1/2} = \boxed{3.7\times10^{-6}\,M}$

$ZnCO_3(s) \rightleftharpoons Zn^{2+} + CO_3^{2-}$

$Zn^{2+} + 4NH_3 \rightarrow Zn(NH_3)_4^{2+}$

$ZnCO_3(s) + 4NH_3(aq) \rightleftharpoons Zn(NH_3)_4^{2+}(aq) + CO_3^{2-}(aq)$

1.0M ~~−4x~~ ignore +X +X

(b) In 1.0M NH_3 $K_f = 4.1\times10^{8}$ $Zn(NH_3)_4^{2+}$

$K_c = 1.4\times10^{-11} \times 4.1\times10^{8} = 5.74\times10^{-3}$

$$5.74\times10^{-3} = \frac{[Zn(NH_3)_4^{2+}][CO_3^{2-}]}{[NH_3]^4}$$

$$5.74\times10^{-3} = \frac{[x][x]}{[1.0]^4} = \frac{x^2}{1} = 5.74\times10^{-3}$$

$x^2 = 5.74\times10^{-3}$

$\boxed{x = 7.6\times10^{-2}\,M}$

Prediction of Precipitation

1.) If 2.00mL of 0.200M NaOH are added to 1.00L of 0.100M $CaCl_2$, will a precipitate form? A: no precipitate will form

$Q > K_{sp}$ Precipitation will occur

$NaOH(aq) + CaCl_2(aq) \longrightarrow 2NaCl + Ca(OH)_2(s)$ (NaCl: soluble; $Ca(OH)_2$: potential precipitate)

$Ca(OH)_2(s) \rightleftharpoons Ca^{2+}(aq) + 2OH^-(aq)$

$K_{sp} = [Ca^{2+}][OH^-]^2$ $\quad K_{sp} = 8.0 \times 10^{-6}$

$[Ca^{2+}]$: $0.100\ M\ CaCl_2$ $\quad \frac{0.100\ mol\ CaCl_2}{}\left(\frac{1\ Ca^{2+}}{1\ CaCl_2}\right)(1.00L) = \frac{0.100\ mol\ Ca^{2+}}{1.002L}$

Total volume: $1.00L + 0.002L = 1.002L$

$= [Ca^{2+}] = 0.0998M$

$[OH^-]$: $\left(0.200\ mol\ NaOH\right)\left(\frac{1\ mol\ Na^+}{1\ mol\ NaOH}\right)(0.002L) = \frac{4.00 \times 10^{-4}\ mol\ OH^-}{1.002L} = [OH^-] = 3.992 \times 10^{-4}M$

$Q = [0.0998][3.992 \times 10^{-4}]^2 = 1.59 \times 10^{-8} = Q < K_{sp} = 8.0 \times 10^{-6}$

no precipitation

Prediction of Precipitation

2.) How many grams of solid K_2SO_4 (174.3g/mol) would need to be added to 50.0mL of a 0.0010M $Ca(NO_3)_2$ solution in order to (a) start precipitation and (b) precipitate 99% of the calcium?

soluble ↙ ↙ precipitate

$$K_2SO_4 + Ca(NO_3)_2 \rightarrow 2KNO_3 + CaSO_4$$

$$CaSO_4(s) \rightleftharpoons Ca^{2+}(aq) + SO_4^{2-}(aq) \quad K_{sp} = 9.1 \times 10^{-6}$$

(a) Start precipitation $Q = K_{sp}$

$K_{sp} = [Ca^{2+}][SO_4^{2-}]$ $\quad [Ca^{2+}] = 0.0010M \quad [SO_4^{2-}] = ?$

$9.1 \times 10^{-6} = [0.0010M][X]$

$[X] = [SO_4^{2-}] = 9.1 \times 10^{-3} M$

$$\frac{9.1 \times 10^{-3}\ mol\ SO_4^{2-}}{L} = \frac{9.1 \times 10^{-3}\ mol\ K_2SO_4}{L} \times 0.0500L = 4.55 \times 10^{-4}\ mol$$

$$4.55 \times 10^{-4}\ \cancel{mol} \times \frac{174.3g}{\cancel{mol}} = \boxed{7.93 \times 10^{-2}g}$$

A: 0.079g

Prediction of Precipitation

2.) How many grams of solid K_2SO_4 (174.3g/mol) would need to be added to 50.0mL of a 0.0010M $Ca(NO_3)_2$ solution in order to (a) start precipitation and (b) precipitate 99% of the calcium?

$CaSO_4(s) \rightleftharpoons Ca^{2+}(aq) + SO_4^{2-}(aq)$ $K_{sp} = 9.1 \times 10^{-6}$

$0.0010\,M \times 0.99 = 9.9 \times 10^{-4}\,M$ removed

Remaining $= 0.0010\,M - 9.9 \times 10^{-4}\,M = 1.0 \times 10^{-5}\,M\ Ca^{2+}$ left

$K_{sp} = 9.1 \times 10^{-6} = [Ca^{2+}][SO_4^{2-}]$

$9.1 \times 10^{-6} = [1.0 \times 10^{-5}][x]$

$x = [SO_4^{2-}] = 0.91\,M\ SO_4^{2-} = 0.91\,M\ K_2SO_4$

$\frac{0.91\,mol}{L} \times 0.0500\,L = 4.55 \times 10^{-2}\,\cancel{mol}\left(\frac{174.3\,g}{\cancel{mol}}\right) = \boxed{7.93\,g}$

A: 7.9g

Selective Precipitation

A solution contains 0.050M Mg^{2+} and 0.020M Cu^{2+}. Solid NaOH is added to the solution. K_{sp} $Mg(OH)_2 = 1.8 \times 10^{-11}$; K_{sp} $Cu(OH)_2 = 4.8 \times 10^{-20}$

(a) Which ion will precipitate first? A: Cu^{2+}

$Cu(OH)_2$ $\quad$ $Mg(OH)_2$

$4.8 \times 10^{-20} \ll 1.8 \times 10^{-11}$

(b) What concentration of OH^- is necessary to begin precipitation of each cation.

A: Cu^{2+} needs 1.5×10^{-9}M
Mg^{2+} needs 1.9×10^{-5}M

$Cu(OH)_{2(s)} \rightleftharpoons Cu^{2+}_{(aq)} + 2OH^-_{(aq)}$

0.020M $\quad$ X

$K_{sp} = [Cu^{2+}][OH^-]^2$

$4.8 \times 10^{-20} = [0.020][X]^2$

$2.4 \times 10^{-18} = X^2$

$X = 1.549 \times 10^{-9}$ M $= [OH^-]$

$Mg(OH)_{2(s)} \rightleftharpoons Mg^{2+}_{(aq)} + 2OH^-_{(aq)}$

0.050M $\quad$ X

$K_{sp} = [Mg^{2+}][OH^-]^2$

$1.8 \times 10^{-11} = [0.050][X]^2$

$3.6 \times 10^{-10} = X^2$

$X = 1.897 \times 10^{-5}$ M $= [OH^-]$