Chemistry 432
Problem Set 1
Spring 2019

1. A ball of mass m is tossed into the air at time $t = 0$ with an initial velocity v_0. The ball experiences a constant acceleration $-g$ from the gravitational attraction of the earth (g is a positive number). Take the initial location of the ball to be at $z = 0$.

   (a) Show that the velocity of the ball at time t is given by

   $$v = v_0 - gt$$

   (b) Show that the height of the ball at time t is given by

   $$z = v_0 t - 1/2 g t^2$$

   (c) Use parts "a" and "b" to show that

   $$v^2 = v_0^2 - 2gz.$$

   (d) Use part "c" to show that the total energy of the ball is conserved during its motion.

2. Show that $x(t) = \cos \omega t$ satisfies Newton's second law for the motion of a harmonic oscillator in 1 dimension. Evaluate $x(t)$ and the linear momentum of the oscillator at time $t = 0$. Show that the sum of the potential and kinetic energies of this oscillator is a constant for all times.

3. Consider a particle of mass m in a one-dimensional harmonic well subject to the potential energy $V = (1/2) m \omega^2 x^2$. Show that

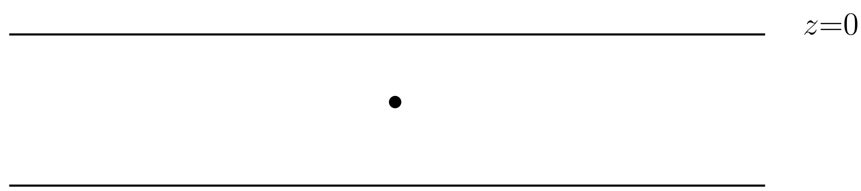
   $$x(t) = A \sin \omega t + B \cos \omega t$$

   is a solution to Newton's second law for the particle. Introduce the boundary conditions

   $$x(0) = x_0 \qquad \text{and} \qquad p(0) = 1$$

   where x_0 is a constant having units of length, to solve for the initial constants A and B. Then show that the sum of the kinetic and potential energies of the particle is independent of time.

4.



A classical particle of mass m and charge q is confined to move between the parallel plates of a capacitor as represented above. The z direction in Cartesian coordinates is taken to be perpendicular to the plates of the capacitor, and the origin of coordinates in the z direction is taken to be the top plate of the capacitor as shown. Between the capacitor plates the electric field, E is a constant, and the potential energy of the charged particle is given by $V(z) = qEz$. Considering only motion in the z direction, give Newton's second law of motion for the particle. Show that the solution to the differential equation associated with Newton's second law is given by

$$z(t) = -\frac{qEt^2}{2m} + c_1 t + c_2$$

where c_1 and c_2 are constants. Evaluate c_1 and c_2 for the initial (boundary) conditions $dz/dt = 0$ at $t = 0$ and $z = 0$ at $t = 0$.

5. The work function of calcium metal is 2.42 eV. Calculate the kinetic energy of electrons emitted from calcium when light of wavelength 454.0 nm shines on the surface.

6. Find the de Broglie wavelength for an electron, a proton and a 100. g bowling ball each having 2. eV of kinetic energy. Find the uncertainty in the position of each particle taking $\Delta p = p$.

7. A one-dimensional electron is bound by a box to a region of space having length 1.0 Å. A second electron is bound by a similar box but of length 2.0 Å. Use the uncertainty principle to estimate the ratio of the zero-point kinetic energies of the two electrons.

8. Consider the expression for the energy of a harmonic oscillator of mass m and angular frequency ω

$$E = \frac{1}{2} m\omega^2 x^2 + \frac{p^2}{2m}.$$

Assuming the uncertainties in the coordinate x and momentum p to be x and p; i.e. write

$$x \approx \Delta x \qquad p \approx \Delta p$$

and using the minimum uncertainty relation

$$\Delta x \Delta p = \frac{\hbar}{2}$$

minimize E with respect to Δx. Show that the energy evaluated at that value of Δx is $E = \hbar\omega/2$. As shown later this semester, the value of the energy at the minimum matches the exact ground state energy of the quantum oscillator.

9. Consider a quantum system with total energy given by the expression

$$E = \frac{p^2}{2m} + \alpha x \qquad 0 \le x < \infty$$

with α a constant. Use the uncertainty principle to estimate the ground-state energy of the system.

10. The expression for the energy of a one-dimensional hydrogen atom composed of a proton at a fixed location in space and a moving electron having mass m, coordinate x and momentum p is given by

$$E = \frac{p^2}{2m} - \frac{K}{x}$$

where K is a collection of constants including the charges and other information unimportant in the current problem. Use the uncertainty principle assuming $\Delta p \sim p$ and $\Delta x \sim x$ and minimize the energy E to estimate the ground-state energy of the one-dimensional hydrogen atom.

11. When light of wavelength 4000 Å shines on the surface of barium metal, electrons of kinetic energy .63 ev are emitted. Calculate the wavelength of light needed so that photoelectrons of de Broglie wavelength 10 Å are emitted from the surface of barium.

12. When light of wavelength 5250Å shines on a particular surface of tungsten metal, the ejected electrons are found to have a de Broglie wavelength of 1.69 $\times 10^{-9}$ m. Calculate the kinetic energies of the ejected electrons if light of frequency 5.00 $\times 10^{14}$ s^{-1} shines on the same surface.

13. The work function of cesium metal is 2.10 eV. Calculate the de Broglie wavelength of photoelectrons emitted from the surface of Cs when light of wavelength 307.1 nm interacts with the surface.

14. When electromagnetic radiation of frequency $\nu = 1.50 \times 10^{15} s^{-1}$ shines on the surface of metallic nickel, electrons are emitted with deBroglie wavelength 11.2Å. When radiation of the same frequency shines on sodium metal, the kinetic energy of the emitted electrons is 3.93 eV. Let ν_{min} be the minimum frequency of light required such that electrons are emitted from the surface of metallic nickel. Calculate the kinetic energy of electrons emitted from the surface of sodium metal if radiation of frequency ν_{min} shines on sodium.

15. When light of a certain wavelength interacts with the surface of samarium (Sm) metal, electrons of deBroglie wavelength $\lambda_{db} = 1.932 \times 10^{-9}$ m are ejected. When light of the same wavelength interacts with the surface of lithium (Li) metal, the deBroglie wavelength of each of the emitted electrons is $\lambda_{db} = 2.719 \times 10^{-9}$ m. Given the work function of lithium is $\phi_{Li} = 2.90$ eV, calculate ϕ_{Sm}, the work function of samarium, and calculate the wavelength of the light that interacts with the metal surfaces.